**Functions**

One of the main goals of this class is to start you along the path to understanding the behavior of the Python code that you will write. In particular, we are interested in understanding how long your program takes to run in terms of the size of the input provide to the program. The key mathematical concept that we will use to model the running time of a program is the *function*. As our first Practice Activity, we will review some basic definitions and properties of functions that you should have learned in high school Algebra.

**Mathematical functions**

A mathematical function is a mapping from a set of inputs to a set of outputs with the property that each input corresponds to exactly one output. The standard mathematical definition of a function has the form

f(p1,p2,...,pn)=body

where f is the name of the function, p1,p2,...,pn is a sequence of *parameters* for the function, and body is an expression involving the parameters p1,p2,...,pn. Supplying values v1,v2,...,vn for the parameters p1,p2,...,pn, we can *call* the function via an expression of the form f(v1,v2,...,vn). To evaluate this expression, we substitute the values vi for the variables pi in expression body and then compute the value of the resulting expression.

For example, consider the function that takes a number and returns the square of the number. Mathematically, this function would have the form square(num)=num2. The function call square(5) would substitute 5 for num and return the value 52=25.

**Python functions**

One nice feature of Python is that function definitions and calls have a form that is very similar to the mathematical form described above. If we temporarily ignore our style guidelines, the square function defined above would be expressed in Python as



A function call in Python is almost identical to a function call in mathematical form. To compute the square of 5 in Python, we would simply use the expression square(5). You should already be comfortable writing simple function definitions in Python if you intend to take this class.

**Polynomial functions**

The most important type of functions that we will consider in the class are polynomial functions. A [polynomial](http://en.wikipedia.org/wiki/Polynomial) is an expression consisting of variables and coefficients, that involves only the operations of addition, subtraction, multiplication, and non-negative integer exponents. A function is a polynomial function if the body of the function is an polynomial whose variables corresponds to the parameters of the function. For example, the function

f(x,y)=xy−x−y+1

is a polynomial function in two variables x and y. A polynomial in one variable is referred to as a *univariate* polynomial. A univariate polynomial in the variable x can be written in the form

cnxn+cn−1xn−1+...+c0

where the coefficients c0,c1,...,cn are numbers. In this form, the integer n is the *degree* of the polynomial. A function of one parameter x is a *linear* function in x if its body is a univariate polynomial of degree at most one in x. Likewise, such a function is a *quadratic* function in x if the body of the function is a univariate polynomial of degree at most two in x. As an example, note that function square defined above is a quadratic function in num. Similar terminology involving cubic, quartic, quintic, etc. applies is the degree of the degree of polynomial corresponding to the body of the function is 3,4,5..., respectively.

**Other important functions**

We will consider several other types of important functions in this class. A [rational](http://en.wikipedia.org/wiki/Rational_function) function is a function whose body is the ratio of two polynomials (i.e; both the numerator and denominator are polynomials). For example, the function

f(x)=x21+x

is a rational function in x since its body is the ratio of the polynomials x2 and 1+x. Another common class of functions is the *exponential* functions of the form f(x)=cx where c is a number. A related class of function is the *logarithmic* functions. The logarithm of a number is the exponent to which another fixed value, the base, must be raised to produce that number. Logarithmic functions are typically written in the form g(y)=logc⁡(y) where c is the base of the logarithm. For example, log2⁡(32) is 5 since 25=32.

If the notion of the logarithm is new to you, a simple method for understanding the behavior of the log function is to note that it is the inverse of a corresponding exponential function. Two functions f and g are *inverses* if they satisfy the equation g(f(x))=x. For exponentials and logarithms, these functions satisfy logc(cx)=x. We will discuss exponentials and logarithms more later in the class.